Relativistic description of the exclusive rare radiative decays of B mesons

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Abstract

The exclusive rare radiative B-decays are studied in the framework of the relativistic quark model based on the quasipotential approach in quantum field theory. The large recoil momentum of the final vector meson allows for the expansion of the decay form factor in inverse powers of the b-quark mass. This considerably simplifies the analysis of these decays. The $1/m_b$ expansion is carried out up to the second order. The form factor of $B \to K^*\gamma$ decay is found to be $F_1^{B\to K^*\gamma}(0) = 0.32 \pm 0.03$ and it leads to $BR(B \to K^*\gamma) = (4.5 \pm 1.5) \times 10^{-5}$, which is in agreement with the recent CLEO data. The form factors of the decays $B \to \rho\gamma$, $B_s \to \phi\gamma$ and $B_s \to K^*\gamma$ are also considered. The relation between rare radiative and semileptonic B-decays into light vector meson is discussed.
PACS number(s): 12.39Ki, 13.20He, 13.40Hq

1 Introduction

Rare radiative decays of B-mesons represent an important test of the standard model of electroweak interactions. These transitions are induced by flavour changing neutral currents and thus they are sensitive probes of new physics (see e. g. [1]). Such decays are governed by one-loop (penguin) diagrams with the main contribution from virtual top quark and W boson. Therefore, they provide valuable information about the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{td}, V_{ts} and V_{tb}, and about the top quark mass. The first exclusive decay $B \to K^*\gamma$ has been observed by CLEO [2]. The measured branching ratio is $BR(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$. Also the inclusive decay rate $B \to X_s\gamma$, obtained from the measurement of the photon energy spectrum, has been reported by CLEO: $BR(B \to X_s\gamma) = (2.32 \pm 0.51 \pm 0.32 \pm 0.20) \times 10^{-4}$ [3].

Theoretical analysis of rare radiative decays is based on the effective Hamiltonian, which is obtained by integrating out the heavy particles. The renormalization of the effective Hamiltonian coefficients has been calculated to leading order [4,5]. It turns out that renormalization effects play an important role. They increase the decay amplitude approximately by a factor of two [4]. Some of the next-to-leading order corrections are also known [6,7]. The hadronic matrix elements of the effective Hamiltonian for inclusive rare radiative decays have been calculated in the framework of the heavy quark effective theory (HQET) [8]. It has been shown that the leading term in $1/m_b$ expansion corresponds to the parton model prediction and the nonperturbative corrections are suppressed by $1/m_b^2$ factor [9,10]. For the calculation of the exclusive decay rates it is necessary to know the relevant hadronic form factors. In the case of $B \to K^*\gamma$ decay only one hadronic form factor $F_1(0)$ contributes. It has been calculated in the framework of nonrelativistic quark model [11] and QCD sum rules [12–14]. In [15] the heavy quark mass limit has been applied to exclusive $B \to K^*\gamma$ decay. However, the s-quark in the final K^* meson is not heavy. Its mass is of order of $\bar{\Lambda}$ parameter, which determines the scale of $1/m_Q$ corrections in HQET [8]. Thus very substantial corrections to this limit come from the whole series in $1/m_s$. Nevertheless, the ideas of heavy quark expansion can be applied to the exclusive $B \to K^*\gamma$ decay. From kinematical analysis it follows that the final K^* meson bears large relativistic recoil momentum $|\mathbf{\Delta}|$ of order of $m_b/2$ and the energy of the same order. So it is possible to expand the matrix element of the effective Hamiltonian both in inverse powers of b-quark mass from the initial B meson and in inverse powers of the recoil momentum $|\mathbf{\Delta}|$ of the final K^* meson. As a result the expansion in powers of $1/m_b$ arises. The aim of this paper is

to realize such expansion in the framework of the relativistic quark model. We show that this expansion considerably simplifies the analysis of exclusive rare radiative decays of B mesons and reduce the model dependence of the result. The leading, first and second order terms of the $1/m_b$ expansion are calculated. It is necessary to note that rare radiative decays of B mesons require the completely relativistic treatment, because the recoil momentum of final vector meson is highly relativistic.

Our relativistic quark model is based on the quasipotential approach in quantum field theory with the specific choice of the $q\bar{q}$ potential. It provides a consistent scheme for calculation of all relativistic corrections at a given order of v^2/c^2 and allows for the heavy quark $1/m_Q$ expansion. This model has been applied for the calculations of meson mass spectra [16], radiative decay widths [17], pseudoscalar decay constants [18], semileptonic [19] and nonleptonic [20] decay rates. The heavy quark $1/m_Q$ expansion in our model for the heavy-to-heavy semileptonic transitions has been developed in [21] up to $1/m_Q^2$ order. The results are in agreement with the model independent predictions of HQET [8]. The rare decay $B \to K^*\gamma$ has been considered in our model in [22]. Here we refine our previous analysis with more complete account of relativistic effects and using $1/m_b$ expansion. We also consider some other exclusive radiative decays, including the decay $B_s \to \phi\gamma$ and the CKM-suppressed decays $B \to \rho\gamma$ and $B_s \to K^*\gamma$.

The paper is organized as follows. In Sect. 2 we briefly review the theoretical predictions for the inclusive rare radiative B-decays and define the form factor F_1, which governs the exclusive decays. The relativistic quark model is described in Sect. 3, and in Sect. 4 it is applied for the calculation of the rare radiative decay form factor F_1. The $1/m_b$ expansion for this form factor is carried out in Sect. 5. Our numerical results for the form factors of the decays $B \to K^*\gamma$, $B_s \to \phi\gamma$, $B \to \rho\gamma$ and $B_s \to K^*\gamma$ are presented in Sect. 6. We also discuss the relations between rare radiative and semileptonic B-decays into light vector mesons. Sect. 7 contains our conclusions.

2 Inclusive and exclusive rare radiative B-decays

In the standard model, B-decays are described by the effective Hamiltonian, obtained by integrating out the top quark and W boson and using the Wilson expansion [4]. For the case of $b \to s$ transition:

$$H_{eff}(b \to s) = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{j=1}^{8} C_j(\mu) O_j(\mu), \tag{1}$$

where V_{ij} are the corresponding CKM matrix elements, $\{O_j\}$ are a complete set of renormalized dimension six operators involving light fields, which govern $b \to s$ transitions. They consist of six four-quark operators $O_j \quad (j = 1, \ldots, 6)$, which determine the non-leptonic B-decay rates, the electromagnetic dipole operator O_7

$$O_7 = \frac{e}{16\pi^2} \bar{s}\sigma^{\mu\nu}(m_b P_R + m_s P_L) b F_{\mu\nu}, \qquad P_{R,L} = (1 \pm \gamma_5)/2, \tag{2}$$

and the chromomagnetic dipole operator O_8, which are responsible for the rare B-decays $B \to X_s\gamma$ and $b \to s + g$, respectively [4]. The Wilson coefficients $C_j(\mu)$ are evaluated perturbatively at the W scale and then they are evolved down to the renormalization scale $\mu \sim m_b$ by the renormalization group equations. The coefficient of the magnetic operator, C_7, has been calculated to leading logarithmic order [4,5]. The next-to-leading order corrections to the anomalous dimension matrix are also partially known [6].

The dominant contribution to the inclusive decay width $\Gamma(B \to X_s\gamma)$ comes from the magnetic moment term $C_7(\mu)O_7(\mu)$. Thus it is necessary to calculate the hadronic matrix element of this operator. Recently it has been observed that the matrix elements of inclusive decays of hadrons containing a heavy quark Q allow for a systematic expansion in powers of $1/m_Q$ [9]. The leading order term of this expansion reproduces the parton model rate and the nonperturbative corrections appear only at the second order of $1/m_Q$ expansion [9]. Therefore, the decay rate for $B \to X_s\gamma$ is [10]

$$\Gamma(B \to X_s\gamma) = \frac{G_F^2 \alpha m_b^5}{32\pi^4} |V_{ts}^* V_{tb}|^2 |C_7(m_b)|^2 \left(1 + \frac{m_s^2}{m_b^2}\right) \left(1 - \frac{m_s^2}{m_b^2}\right)^3$$

$$\times\left(1+\frac{\lambda_1-9\rho\lambda_2}{2m_b^2}+O\left(\frac{1}{m_b^3}\right)\right), \tag{3}$$

with $\rho=(3m_b+5m_s)/(3m_b-3m_s)$. The parameter λ_1 is related to the kinetic energy of the b-quark inside B meson and λ_2 is related to the $B-B^*$ mass splitting [8]. This rate is strongly dependent on the value of b-quark mass m_b, which depends on the definition. So it is more convinient to connect $\Gamma(B\to X_s\gamma)$ with the experimentally observed semileptonic decay rate.

The branching ratio $BR(B\to X_s\gamma)$ can be expressed in terms of the inclusive semileptonic branching ratio $BR(B\to X\ell\nu_\ell)$ as [7]

$$BR(B\to X_s\gamma)=6\frac{\alpha}{\pi}\frac{|V_{ts}^*V_{tb}|^2}{|V_{cb}|^2}\frac{|C_7(m_b)|^2K(m_b)}{g(m_c/m_b)\left(1-\frac{2}{3}\frac{\alpha_s}{\pi}f(m_c/m_b)\right)}BR(B\to X\ell\nu_\ell), \tag{4}$$

where $g(r)$ is the phase-space factor for $\Gamma(b\to c\ell\nu_\ell)$: $g(r)=1-8r^2+8r^6-r^8-24r^4\ln(r)$, and experimental value $BR(B\to X\ell\nu_\ell)\approx 10.5\%$ is used for the semileptonic branching fraction. The function $f(r)$ accounts for QCD corrections to the semileptonic decay [23] and, for a typical quark mass ratio of $r=0.35\pm0.05$, it has the value $f(r)=2.37\mp0.13$. The factor $K(m)$ contains $O(\alpha_s)$ corrections to the $B\to X_s\gamma$ rate, due to the QCD bremsstrahlung processes [7]. The resulting branching ratio is [7,24]

$$BR(B\to X_s\gamma)=(3.0\pm1.2)\times10^{-4}, \tag{5}$$

for m_t in the range $100<m_t<200$ GeV.

This result agrees with a recent data on the inclusive decay rate by CLEO collaboration [3]:

$$BR(B\to X_s\gamma)=(2.32\pm0.51\pm0.32\pm0.20)\times10^{-4}. \tag{6}$$

In what follows we shall discuss the exclusive rare radiative decays of B mesons, like $B_{u,d}\to K^*\gamma$, $B_s\to\phi\gamma$ and the CKM-suppressed chanels $B_{u,d}\to\rho\gamma$, $B_s\to K^*\gamma$. We denote all these modes by $B\to V\gamma$, where B is $B_{u,d}$ or B_s meson and V is K^*, ϕ or ρ. The main contribution to the exclusive $B\to V\gamma$ decay amplitude comes from the magnetic moment operator O_7. The relevant matrix element has the covariant decomposition [11]:

$$\begin{aligned}\langle V(p_V,e)|\bar{f}i\sigma_{\mu\nu}q^\nu P_R b|B(p_B)\rangle &= i\epsilon_{\mu\nu\rho\sigma}e^{*\nu}p_B^\rho p_V^\sigma F_1(q^2)\\ &\quad+\left[e_\mu^*(M_B^2-M_V^2)-(e^*\cdot q)(p_B+p_V)_\mu\right]G_2(q^2),\end{aligned} \tag{7}$$

where $G_2(0)=F_1(0)/2$, e is a polarization vector of final vector meson, $q=p_B-p_V$ denotes the four-momenta of the emitted photon. The exclusive decay rate is given by

$$\Gamma(B\to K^*\gamma)=\frac{G_F^2\alpha}{32\pi^4}|V_{ts}^*V_{tb}|^2|F_1(0)|^2|C_7(m_b)|^2(m_b^2+m_s^2)\frac{(M_B^2-M_{K^*}^2)^3}{M_B^3}, \tag{8}$$

and the analogous expressions can be written for $\Gamma(B_s\to\phi\gamma)$, $\Gamma(B\to\rho\gamma)$ and $\Gamma(B_s\to K^*\gamma)$. Then the exclusive branching ratio $BR(B\to K^*\gamma)$ is

$$BR(B\to K^*\gamma)=R\cdot BR(B\to X_s\gamma), \tag{9}$$

where the ratio of the exclusive to inclusive radiative decay rates is determined by (3), (8) to be:

$$R\equiv\frac{\Gamma(B\to K^*\gamma)}{\Gamma(B\to X_s\gamma)}=\frac{\left(1-M_{K^*}^2/M_B^2\right)^3}{\left(1-m_s^2/m_b^2\right)^3}\frac{M_B^3}{m_b^3}\frac{|F_1(0)|^2}{K(m_b)\left(1+(\lambda_1-9\rho\lambda_2)/(2m_b^2)\right)}. \tag{10}$$

The factor $K(m_b)\approx0.83$ [7], the parameters $\lambda_1=-0.30\pm0.30$ GeV^2 [8,25] and $\lambda_2=0.12\pm0.01$ GeV^2 [8].

We shall calculate the form factor $F_1(0)$ with the account of $1/m_b$ corrections up to the second order in the framework of the relativistic quark model based on the quasipotential method.

3 Relativistic quark model

In the quasipotential approach meson is described by the wave function of the bound quark-antiquark state, which satisfies the quasipotential equation [26] of the Schrödinger type [27]:

$$\left(\frac{b^2(M)}{2\mu_R}-\frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p})=\int\frac{d^3q}{(2\pi)^3}V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q}), \tag{11}$$

where the relativistic reduced mass is

$$\mu_R=\frac{M^4-(m_a^2-m_b^2)^2}{4M^3}; \tag{12}$$

$$b^2(M)=\frac{[M^2-(m_a+m_b)^2][M^2-(m_a-m_b)^2]}{4M^2}, \tag{13}$$

$m_{a,b}$ are the quark masses; M is the meson mass; $\mathbf{p}$ is the relative momentum of quarks. While constructing the kernel of this equation $V(\mathbf{p},\mathbf{q};M)$ — the quasipotential of quark-antiquark interaction — we have assumed that effective interaction is the sum of the one-gluon exchange term with the mixture of long-range vector and scalar linear confining potentials. We have also assumed that at large distances quarks acquire universal nonperturbative anomalous chromomagnetic moments and thus the vector long-range potential contains the Pauli interaction. The quasipotential is defined by [16]:

$$V(\mathbf{p},\mathbf{q},M)=\bar{u}_a(p)\bar{u}_b(-p)\Big\{\frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_a^\mu\gamma_b^\nu+V^V_{conf}(\mathbf{k})\Gamma_a^\mu\Gamma_{b;\mu}+V^S_{conf}(\mathbf{k})\Big\}u_a(q)u_b(-q), \tag{14}$$

where α_S is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator; γ_μ and $u(p)$ are the Dirac matrices and spinors; $\mathbf{k}=\mathbf{p}-\mathbf{q}$; the effective long-range vector vertex is

$$\Gamma_\mu(\mathbf{k})=\gamma_\mu+\frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu, \tag{15}$$

κ is anomalous chromomagnetic quark moment. Vector and scalar confining potentials in the nonrelativistic limit reduce to

$$V^V_{conf}(r)=(1-\varepsilon)(Ar+B),\quad V^S_{conf}(r)=\varepsilon(Ar+B), \tag{16}$$

reproducing $V^{conf}_{nonrel}(r)=V^S_{conf}+V^V_{conf}=Ar+B$, where ε is the mixing coefficient. The explicit expression for the quasipotential with the account of the relativistic corrections of order v^2/c^2 can be found in ref. [16]. All the parameters of our model: quark masses, parameters of linear confining potential A and B, mixing coefficient ε and anomalous chromomagnetic quark moment κ were fixed from the analysis of meson masses [16] and radiative decays [17]. Quark masses: $m_b=4.88$ GeV; $m_c=1.55$ GeV; $m_s=0.50$ GeV; $m_{u,d}=0.33$ GeV and parameters of linear potential: $A=0.18$ GeV^2; $B=-0.30$ GeV have standard values for quark models. The value of mixing coefficient of vector and scalar confining potentials $\varepsilon=-0.9$ has been primarily chosen from the consideration of meson radiative decays, which are very sensitive to the Lorentz-structure of the confining potential: the resulting leading relativistic corrections coming from vector and scalar potentials have opposite signs for the radiative Ml - decays [17]. Universal anomalous chromomagnetic moment of quark $\kappa=-1$ has been fixed from the analysis of the fine splitting of heavy quarkonia ^3P_J- states [16].

Recently we have considered the expansion of the matrix elements of weak heavy quark currents between pseudoscalar and vector meson states up to the second order in inverse powers of the heavy quark masses [21]. It has been found that the general structure of leading, subleading and second order $1/m_Q$ corrections in our relativistic model is in accord with the predictions of HQET. The heavy quark symmetry and QCD impose rigid constraints on the parameters of the long-range potential of our model. The analysis of the first order corrections [21] allowed to fix the value of effective long-range anomalous chromomagnetic moment of quarks $\kappa=-1$, which coincides with the result, obtained from the mass

spectra [16]. The mixing parameter of vector and scalar confining potentials has been found from the comparison of the second order corrections to be $\varepsilon = -1$. This value is very close to the previous one $\varepsilon = -0.9$ determined from radiative decays of mesons [17]. Therefore, we have got QCD and heavy quark symmetry motivation for the choice of the main parameters of our model. The found values of ε and κ imply that confining quark-antiquark potential has predominantly Lorentz-vector structure, while the scalar potential is anticonfining and helps to reproduce the initial nonrelativistic potential.

4 Calculation of the rare radiative decay form factor

The amplitude of the exclusive $B \to V\gamma$ decay is proportional to the hadronic matrix element (7) of the magnetic moment operator O_7. Thus it is necessary to calculate the transition form factor $F_1(0)$.

The matrix element of the local current J between bound states in the quasipotential method has the form [28]:

$$\langle V|J_\mu(0)|B\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6} \bar{\Psi}_V(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi_B(\mathbf{q}), \tag{17}$$

where $\Gamma_\mu(\mathbf{p},\mathbf{q})$ is the two-particle vertex function and $\Psi_{V,B}$ are the meson wave functions projected onto the positive energy states of quarks.

In the case of rare radiative decays $J_\mu = \bar{f} i\sigma_{\mu\nu}k^\nu P_R b$ and in order to calculate its matrix element between meson states it is necessary to consider the contributions to Γ from Figs. 1 and 2. Thus the vertex functions look like

$$\Gamma_\mu^{(1)}(\mathbf{p},\mathbf{q}) = \bar{u}_f(p_1)\frac{i}{2}\sigma_{\mu\nu}k^\nu(1+\gamma^5)u_b(q_1)(2\pi)^3\delta(\mathbf{p}_2-\mathbf{q}_2), \tag{18}$$

and

$$\begin{aligned}\Gamma_\mu^{(2)}(\mathbf{p},\mathbf{q}) \quad = \quad & \bar{u}_f(p_1)\bar{u}_q(p_2)\frac{1}{2}\Big\{ i\sigma_{1\mu\nu}(1+\gamma_1^5)\frac{\Lambda_b^{(-)}(k_1)}{\varepsilon_b(k_1)+\varepsilon_b(p_1)}\gamma_1^0 V(\mathbf{p}_2-\mathbf{q}_2)\\ & +V(\mathbf{p}_2-\mathbf{q}_2)\frac{\Lambda_f^{(-)}(k_1')}{\varepsilon_f(k_1')+\varepsilon_f(q_1)}\gamma_1^0 i\sigma_{1\mu\nu}(1+\gamma_1^5)\Big\} u_b(q_1)u_q(q_2),\end{aligned} \tag{19}$$

where $\mathbf{k}_1 = \mathbf{p}_1 - \mathbf{\Delta}$; $\quad \mathbf{k}_1' = \mathbf{q}_1 + \mathbf{\Delta}$; $\quad \mathbf{\Delta} = \mathbf{p}_B - \mathbf{p}_V$; $\quad \varepsilon(p) = (m^2+\mathbf{p}^2)^{1/2}$;

$$\Lambda^{(-)}(p) = \frac{\varepsilon(p) - \big(m\gamma^0 + \gamma^0(\boldsymbol{\gamma}\mathbf{p})\big)}{2\varepsilon(p)}.$$

Note that the contribution $\Gamma^{(2)}$ is the consequence of the projection onto the positive-energy states. The form of the relativistic corrections resulting from the vertex function $\Gamma^{(2)}$ is explicitly dependent on the Lorentz-structure of $q\bar{q}$-interaction.

It is convenient to consider the decay $B \to V\gamma$ in the B meson rest frame. Then the recoil momentum $\mathbf{\Delta}$ of vector meson V is highly relativistic ($|\mathbf{\Delta}| \approx M_B/2$). The wave function of the meson moving with the momentum $\mathbf{\Delta}$ is connected with the wave function at rest by the transformation [28]

$$\Psi_{V\,\mathbf{\Delta}}(\mathbf{p}) = D_f^{1/2}(R_{L\mathbf{\Delta}}^W)D_q^{1/2}(R_{L\mathbf{\Delta}}^W)\Psi_{V\,\mathbf{0}}(\mathbf{p}), \tag{20}$$

where $D^{1/2}(R)$ is the well-known rotation matrix and R^W is the Wigner rotation.

The meson wave functions in the rest frame have been calculated by numerical solution of the quasipotential equation (11) [29]. However, it is more convenient to use analytical expressions for meson wave functions. The examination of numerical results for the ground state wave functions of mesons containing at least one light quark has shown that they can be well approximated by the Gaussian functions

$$\Psi_M(\mathbf{p}) \equiv \Psi_{M\,\mathbf{0}}(\mathbf{p}) = \Big(\frac{4\pi}{\beta_M^2}\Big)^{3/4}\exp\Big(-\frac{\mathbf{p}^2}{2\beta_M^2}\Big), \tag{21}$$

with the deviation less than 5%.

The parameters are

$$\beta_B = 0.41 \text{ GeV}; \qquad \beta_{K^*} = 0.33 \text{ GeV}; \qquad \beta_\rho = 0.31 \text{ GeV};$$
$$\beta_{B_s} = 0.46 \text{ GeV}; \qquad \beta_\phi = 0.36 \text{ GeV}. \tag{22}$$

Substituting the vertex functions (18), (19), with the account of wave function transformation of vector meson (20), in the matrix element (17) we get for the form factor F_1 the following expression:

$$F_1(0) = F_1^{(1)}(0) + \varepsilon F_1^{S(2)}(0) + (1-\varepsilon) F_1^{V(2)}(0), \tag{23}$$

$$\begin{aligned} F_1^{(1)}(0) &= \sqrt{\frac{E_V}{M_B}} \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_V\left(\mathbf{p} + \frac{2\varepsilon_q}{E_V+M_V}\boldsymbol{\Delta}\right) \sqrt{\frac{\varepsilon_f(p+\Delta)+m_f}{2\varepsilon_f(p+\Delta)}} \sqrt{\frac{\varepsilon_b(p)+m_b}{2\varepsilon_b(p)}} \\ &\times \Bigg\{ 1 + \frac{p_z^2 + (\mathbf{p}\boldsymbol{\Delta})}{(\varepsilon_f(p+\Delta)+m_f)(\varepsilon_b(p)+m_b)} + (M_B - E_V)\Bigg[\frac{1}{\varepsilon_f(p+\Delta)+m_f} \\ &+ \frac{(\mathbf{p}\boldsymbol{\Delta})}{\boldsymbol{\Delta}^2}\left(\frac{1}{\varepsilon_f(p+\Delta)+m_f} + \frac{1}{\varepsilon_b(p)+m_b}\right) \\ &- \frac{p_x^2+p_y^2}{2(E_V+M_V)}\Bigg(\frac{1}{\varepsilon_q(p)+m_q}\left(\frac{1}{\varepsilon_b(p)+m_b} - \frac{1}{\varepsilon_f(p+\Delta)+m_f}\right) \\ &+ \frac{4}{(\varepsilon_f(p)+m_f)(\varepsilon_b(p)+m_b)}\Bigg)\Bigg]\Bigg\}\Psi_B(\mathbf{p}), \end{aligned} \tag{24}$$

$$\begin{aligned} F_1^{S(2)}(0) &= \sqrt{\frac{E_V}{M_B}} \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_V\left(\mathbf{p} + \frac{2\varepsilon_q}{E_V+M_V}\boldsymbol{\Delta}\right) \sqrt{\frac{\varepsilon_f(p+\Delta)+m_f}{2\varepsilon_f(p+\Delta)}} \\ &\times \Bigg\{ \frac{\varepsilon_f(\Delta)-m_f}{2\varepsilon_f(\Delta)(\varepsilon_f(\Delta)+m_f)}\left(1 + \frac{M_B-E_V}{2m_b}\frac{(\mathbf{p}\boldsymbol{\Delta})}{\boldsymbol{\Delta}^2}\right) \\ &\times \left(M_V - \varepsilon_f\left(p + \frac{2\varepsilon_q}{E_V+M_V}\Delta\right) - \varepsilon_q\left(p + \frac{2\varepsilon_q}{E_V+M_V}\Delta\right)\right) - (M_B-E_V)\frac{(\mathbf{p}\boldsymbol{\Delta})}{\boldsymbol{\Delta}^2} \\ &\times \Bigg(\frac{M_B + M_V - \varepsilon_b(p) - \varepsilon_q(p) - \varepsilon_f\left(p + \frac{2\varepsilon_q}{E_V+M_V}\Delta\right) - \varepsilon_q\left(p + \frac{2\varepsilon_q}{E_V+M_V}\Delta\right)}{2m_b(\varepsilon_b(\Delta)+m_b)} \\ &+ \frac{M_B - M_V - \varepsilon_b(p) - \varepsilon_q(p) + \varepsilon_f\left(p + \frac{2\varepsilon_q}{E_V+M_V}\Delta\right) + \varepsilon_q\left(p + \frac{2\varepsilon_q}{E_V+M_V}\Delta\right)}{2\varepsilon_f(\Delta)(\varepsilon_f(\Delta)+m_f)}\Bigg)\Bigg\}\Psi_B(\mathbf{p}), \end{aligned} \tag{25}$$

$$\begin{aligned} F_1^{V(2)}(0) &= \sqrt{\frac{E_V}{M_V}} \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_V\left(\mathbf{p} + \frac{2\varepsilon_q}{E_V+M_V}\boldsymbol{\Delta}\right) \sqrt{\frac{\varepsilon_f(p+\Delta)+m_f}{2\varepsilon_f(p+\Delta)}} \\ &\times \Bigg\{ \frac{\varepsilon_f(\Delta)-m_f}{2\varepsilon_f(\Delta)(\varepsilon_f(\Delta)+m_f)}\left(1 + \frac{M_B-E_V}{2m_b}\frac{(\mathbf{p}\boldsymbol{\Delta})}{\boldsymbol{\Delta}^2}\right) \\ &\times \left(M_V - \varepsilon_f\left(p + \frac{2\varepsilon_q}{E_V+M_V}\Delta\right) - \varepsilon_q\left(p + \frac{2\varepsilon_q}{E_V+M_V}\Delta\right)\right) + (M_B-E_V)\frac{(\mathbf{p}\boldsymbol{\Delta})}{\boldsymbol{\Delta}^2} \\ &\times \Bigg(\frac{M_V - \varepsilon_f\left(p + 2\varepsilon_q E_V + M_V\Delta\right) - \varepsilon_q\left(p + \frac{2\varepsilon_q}{E_V+M_V}\Delta\right)}{2m_b(\varepsilon_b(\Delta)+m_b)} + \frac{M_B - \varepsilon_b(p) - \varepsilon_q(p)}{2\varepsilon_f(\Delta)(\varepsilon_f(\Delta)+m_f)} \\ &+ \frac{1+\kappa}{2(\varepsilon_q(p)+m_q)}\left(\frac{1}{\varepsilon_b(\Delta)+m_b} - \frac{1}{\varepsilon_f(\Delta)}\right)\Bigg(M_B - M_V - \varepsilon_b(p) \end{aligned}$$

$$+\varepsilon_f\left(p+\frac{2\varepsilon_q}{E_V+M_V}\Delta\right)+\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_V+M_V}\Delta\right)\Bigg)\Bigg)\Bigg\}\Psi_B(\mathbf{p}), \tag{26}$$

where the superscripts "(1)" and "(2)" correspond to Figs. 1 and 2, S and V — to the scalar and vector potentials of $q\bar{q}$-interaction;

$$|\mathbf{\Delta}|=\frac{M_B^2-M_V^2}{2M_B}; \qquad E_V=\frac{M_B^2+M_V^2}{2M_B}; \tag{27}$$

and z-axis is chosen in the direction of $\mathbf{\Delta}$. The contributions to the form factor coming from Fig. 2 are proportional to the binding energy of B or V mesons. Taking into account that in our model $m_b+m_d\simeq M_b$; $m_b+m_s\simeq M_{B_s}$; $m_s+m_d\simeq M_{K^*}$; $2m_s\simeq M_\phi$; $2m_d\simeq M_\rho$ and $|\mathbf{\Delta}|^2\simeq m_b^2/4\gg\langle\mathbf{p}^2\rangle$ we have neglected the terms proportional to the product of binding energies and ratios $\mathbf{p}^2/\varepsilon_f^3(\Delta)$, $\mathbf{p}^2/\varepsilon_b^3(\Delta)$. The omitted terms are of order $1/m_b^3$.

The expressions (23)–(26) for the form factor $F_1(0)$ differ from our previous results [22] in the argument of the wave function of final vector meson V. In [22] we have used for simplicity the nonrelativistic limit $\mathbf{p}+\frac{m_q}{M_V}\mathbf{\Delta}$ for this argument, taking into account the approximation of the wave functions by Gaussians. However, our recent analysis of the $1/m_Q$ corrections to the semileptonic decays [21] has shown, that such approximation is not completely adequate. It is necessary to take relativistic expression $\mathbf{p}+\frac{2\varepsilon_q}{E_V+M_V}\mathbf{\Delta}$ in the argument of the wave function, what is done in eqs.(23)–(26).

5 $1/m_b$ expansion for the form factor $F_1(0)$

At present considerable theoretical interest is attracted to HQET. This theory is based on QCD and heavy quark expansion. Additional heavy quark symmetries (for a review see e. g. [8]) arise in the limit of infinitely heavy quark mass. HQET provides a systematic expansion in inverse powers of the heavy quark mass of hadronic matrix elements between mesons with one heavy and one light quarks. In the case of exclusive heavy-to-heavy transitions heavy quark symmetries considerably reduce the number of independent form factors in each order of $1/m_Q$ expansion. This simplifies the theoretical analysis of such decays. In exclusive heavy-to-light meson transitions (such as the $B\to V\gamma$ decays) the predictive power of HQET is essentialy less. Only the relations between various form factors of the semileptonic and rare B-decays are imposed in the limit of infinitely heavy b-quark mass [30]. However, as it has been already mentioned in the introduction, the large value of the recoil momentum of final vector meson $|\mathbf{\Delta}|\sim m_b/2$ allows the expansion in powers of $1/m_b$ for the radiative rare decay form factor $F_1(0)$. This expansion leads to considerable simplification of the formulae for $F_1(0)$. It has been already partly used by us in derivation of (23)–(26). The large value of recoil momentum $|\mathbf{\Delta}|$ allowed us to neglect $\mathbf{p}^2$ in comparison with $\mathbf{\Delta}^2$ in the quark energy $\varepsilon_f(p+\Delta)$ in final meson in the expressions for $F_1^{S(2)}$ and $F_1^{V(2)}$. Thus we were able to perform one of the integrations in the current matrix element (17) using quasipotential equation as in the case of the heavy final meson [17,19,28]. As a result, we get more compact formulae. In this section for the sake of consistency we carry out the complete expansion of (23)–(26) in inverse powers of b-quark mass.

In HQET the mass of B meson has the following expansion in $1/m_b$ [8]

$$M_B=m_b+\bar{\Lambda}+\frac{\Delta m_B^2}{2m_b}+O\left(\frac{1}{m_b^2}\right), \tag{28}$$

where parameter $\bar{\Lambda}$ is the difference between the meson and quark masses in the limit of infinitely heavy quark mass. In our model $\bar{\Lambda}$ is equal to the mean value of light quark energy inside the heavy meson $\bar{\Lambda}=\langle\varepsilon_q\rangle_B\approx 0.54$ GeV [21]. Δm_B^2 arises from the first-order power corrections to the HQET Lagrangian and has the form [8]:

$$\Delta m_B^2=-\lambda_1-3\lambda_2. \tag{29}$$

The parameter λ_1 results from the mass shift due to the kinetic operator, while λ_2 parameterizes the chromomagnetic interaction [8]. The value of spin-symmetry breaking parameter λ_2 is related to the vector-pseudoscalar mass splitting

$$\lambda_2 \approx \frac{1}{4}(M_{B^*}^2 - M_B^2) = 0.12 \pm 0.01 \text{ GeV}^2.$$

The parameter λ_1 is not directly connected with observable quantities. Theoretical predictions for it vary in a wide range: $\lambda_1 = -0.30 \pm 0.30$ GeV^2 [8,25].

In the limit $m_Q \to \infty$, meson wave functions become independent of the flavour of heavy quark. Thus the Gaussian parameter β_B in (21) should have the following expansion [21]

$$\beta_B = \beta - \frac{\Delta\beta^2}{m_b} + O\left(\frac{1}{m_b^2}\right), \qquad \beta \approx 0.42 \text{ GeV}, \tag{30}$$

where the second term breaks the flavour symmetry and in our model is equal to $\Delta\beta^2 \approx 0.045$ GeV^2 [21].

Substituting (28) in (27) we get the $1/m_b$ expansion of the recoil momentum and the energy of final vector meson:

$$\begin{aligned} |\mathbf{\Delta}| &= \frac{m_b}{2}\left(1 + \frac{1}{m_b}\bar{\Lambda} + \frac{1}{m_b^2}\left(\frac{\Delta m_B^2}{2} - M_V^2\right)\right) + O\left(\frac{1}{m_b^2}\right), \\ E_V &= \frac{m_b}{2}\left(1 + \frac{1}{m_b}\bar{\Lambda} + \frac{1}{m_b^2}\left(\frac{\Delta m_B^2}{2} + M_V^2\right)\right) + O\left(\frac{1}{m_b^2}\right). \end{aligned} \tag{31}$$

Now we use the Gaussian approximation for the wave functions (21). Then shifting the integration variable $\mathbf{p}$ in (23)–(26) by $-\frac{\varepsilon_q}{E_V + M_V}\mathbf{\Delta}$, we can factor out the $\mathbf{\Delta}$ dependence of the meson wave function overlap in form factor F_1. The result can be written in the form

$$F_1(0) = \mathcal{F}_1(\mathbf{\Delta}^2)\exp(-\zeta\mathbf{\Delta}^2), \tag{32}$$

where $|\mathbf{\Delta}|$ is given by (27) and

$$\zeta\mathbf{\Delta}^2 = \frac{2\tilde{\Lambda}^2\mathbf{\Delta}^2(\beta_B^2 + \beta_V^2)(E_V + M_V)^2}{=}\frac{2\tilde{\Lambda}^2}{(\beta_B^2 + \beta_V^2)}\left(\frac{M_B - M_V}{M_B + M_V}\right)^2, \tag{33}$$

here $\tilde{\Lambda}$ is equal to the mean value of light quark energy between heavy and light meson states:

$$\tilde{\Lambda} = \langle \varepsilon_q \rangle. \tag{34}$$

Expanding (33) in powers of $1/m_b$ we get

$$\zeta\mathbf{\Delta}^2 = \frac{\tilde{\Lambda}^2}{\beta^2}\eta\left(1 - 4\frac{M_V}{m_b}\right) + O\left(\frac{1}{m_b^2}\right), \tag{35}$$

where $\eta = \frac{2\beta_B^2}{\beta_B^2 + \beta_V^2}$. We see that the first term in this expansion is large. Really, even in the case of the lightest final meson ρ: $4M_\rho/m_b \approx 0.63$. The value of this correction to the form factor $F_1(0)$ is also increased by the exponentiating in (32). Therefore, we conclude that the first order correction in $1/m_b$ expansion of $F_1(0)$, arising from the meson wave function overlap, is large. Thus, in the following, we use unexpanded expression (33) in the exponential of the form factor $F_1(0)$ in (32).

In contrast to the meson wave function overlap the factor $\mathcal{F}_1(\mathbf{\Delta}^2)$ in (32) has a well defined $1/m_b$ expansion. First and second order corrections are small. Substituting the Gaussian wave functions (21) in the expressions for the form factor (23)–(26), with the value of anomalous chromomagnetic quark

moment $\kappa=-1$, and using (32) and the expansions (28)–(31), we get up to the second order in $1/m_b$ expansion:

$$\begin{aligned}
\mathcal{F}_1(\mathbf{\Delta}^2) &= \mathcal{F}_1^{(1)}(\mathbf{\Delta}^2)+\varepsilon\mathcal{F}_1^{S(2)}(\mathbf{\Delta}^2)+(1-\varepsilon)\mathcal{F}_1^{V(2)}(\mathbf{\Delta}^2); &(36)\\
\mathcal{F}_1^{(1)}(\mathbf{\Delta}) &= N\Bigg(1-\frac{1}{2m_b}\left(\tilde\Lambda\eta-\left\langle\mathbf{p}^2\left(\frac{1}{2}\frac{1}{\bar\varepsilon_q+m_q}-\frac{1}{3}\frac{1}{\bar\varepsilon_f+m_f}\right)\right\rangle\right)\\
&\quad+\frac{1}{2m_b^2}\Bigg(M_V^2-m_f^2-\frac{13}{4}\langle\mathbf{p}^2\rangle-\frac{1}{4}\tilde\Lambda^2\eta^2+\tilde\Lambda\eta(4m_f+2M_V)\\
&\quad+\tilde\Lambda\eta^2\frac{\beta_V^2}{\beta^2}\frac{\Delta\beta^2}{\beta}-\frac{1}{3}\left\langle\frac{\mathbf{p}^2}{\bar\varepsilon_q+m_q}\right\rangle\left(\frac{5}{2}m_f+3M_V+2\bar\Lambda-4\tilde\Lambda\eta\right)\\
&\quad+\frac{1}{3}\left\langle\frac{\mathbf{p}^2}{\bar\varepsilon_f+m_f}\right\rangle(2M_V-m_f)\Bigg)\Bigg); &(37)\\
\mathcal{F}_1^{S(2)}(\mathbf{\Delta}^2) &= N\frac{1}{2}\Bigg(\frac{1}{m_b}\left(1-\frac{1}{m_b}(3m_f+\bar\Lambda+\frac{1}{2}\tilde\Lambda\eta)\right)(M_V-\langle\bar\varepsilon_q\rangle-\langle\bar\varepsilon_f\rangle)\\
&\quad+\frac{\tilde\Lambda\eta}{m_b^2}\Bigg(\frac{1}{2+\sqrt5}(\bar\Lambda+M_V-\langle\bar\varepsilon_f\rangle-2\langle\bar\varepsilon_q\rangle)+2(\bar\Lambda-M_V+\langle\bar\varepsilon_f\rangle)\\
&\quad-\frac{3}{2}\left\langle\frac{\mathbf{p}^2}{\bar\varepsilon_q}\right\rangle-\frac{1}{3}\left(\frac{5}{2}-\frac{1}{2+\sqrt5}\right)\left\langle\frac{\mathbf{p}^2}{\bar\varepsilon_f}\right\rangle\Bigg)\Bigg); &(38)\\
\mathcal{F}_1^{V(2)}(\mathbf{\Delta}^2) &= N\frac{1}{2}\Bigg(\frac{1}{m_b}\left(1-\frac{1}{m_b}(3m_f+\bar\Lambda+\frac{1}{2}\tilde\Lambda\eta)\right)(M_V-\langle\bar\varepsilon_q\rangle-\langle\bar\varepsilon_f\rangle)\\
&\quad-\frac{\tilde\Lambda\eta}{m_b^2}\Bigg(\frac{1}{2+\sqrt5}(M_V-\langle\bar\varepsilon_f\rangle-\langle\bar\varepsilon_q\rangle)+2(\bar\Lambda-\langle\bar\varepsilon_q\rangle)\\
&\quad+\frac{1}{6}\left(1+\frac{2}{2+\sqrt5}\right)\left\langle\frac{\mathbf{p}^2}{\bar\varepsilon_f}\right\rangle-\left(\frac{1}{2}-\frac{1}{3(2+\sqrt5)}\right)\left\langle\frac{\mathbf{p}^2}{\bar\varepsilon_q}\right\rangle\Bigg)\Bigg), &(39)
\end{aligned}$$

where $N=\left(\frac{2\beta_B\beta_V}{\beta_B^2+\beta_V^2}\right)^{3/2}=\left(\frac{\beta_V}{\beta_B}\eta\right)^{3/2}$ is due to the normalization of Gaussian wave functions in (21); $\bar\varepsilon_i=\sqrt{\mathbf{p}^2+m_i^2+\tilde\Lambda^2\eta^2}$ $(i=f,q)$, i. e. the energies of light quarks in final vector meson acquire additional contribution from the recoil momentum. The averaging is taken over the Gaussian wave functions of B and V mesons, so it can be carried out analytically. For example,

$$\langle\bar\varepsilon_i\rangle=\frac{1}{\sqrt\pi}\frac{\bar m_i^2}{\beta_V\sqrt\eta}e^zK_1(z), \tag{40}$$

where $\bar m_i^2=m_i^2+\tilde\Lambda^2\eta^2$ and $K_1(z)$ is the modified Bessel function; $z=\bar m_i^2/2\eta\beta_V^2$. Analogous expressions can be obtained for the other matrix elements in (37)–(39).

Our final result for the rare radiative decay $B\to V\gamma$ form factor $F_1(0)$ is given up to the second order of $1/m_b$ expansion by eqs. (32), (33) and (36)–(39).

6 Results and discussion

Using the parameters (22) of the wave functions (21) in the expressions for the form factor $F_1(0)$ (32),(33) and (36)–(39) we get (for the values of the anomalous chromomagnetic quark moment $\kappa=-1$ and the mixing parameter of vector and scalar confining potentials $\varepsilon=-1$ [21]):

$$\begin{aligned}
F_1^{B\to K^*\gamma}(0) &= 0.32\pm0.03 \qquad F_1^{B\to\rho\gamma}(0) = 0.26\pm0.03\\
F_1^{B_s\to\phi\gamma}(0) &= 0.27\pm0.03 \qquad F_1^{B_s\to K^*\gamma}(0) = 0.23\pm0.02. &(41)
\end{aligned}$$

The theoretical uncertainty in (41) results mostly from the approximation of the wave functions by Gaussians (21) and does not exceed 10% of form factor values. The contributions of higher order terms in $1/m_b$ expansion in (36)–(39) are negligibly small. Thus the unexpanded (23)–(26) and expanded (41) values of form factors differ unessentially. This conclusion is confirmed by numerical analysis.

Our results (41) for the form factor values are in good agreement with recent calculations within the light-cone QCD sum rule [13] and hybrid sum rule [14] approaches. The comparison of predictions is given in Table 1. All results agree within errors.

The calculated value of form factor $F_1^{B\to K^*\gamma}$ in (41) yields for the ratio (10) of exclusive to inclusive radiative decay widths:

$$R(B\to K^*\gamma)\equiv\frac{\Gamma(B\to K^*\gamma)}{\Gamma(B\to X_s\gamma)}=(15\pm 3)\%. \tag{42}$$

Combining this result with the QCD-improved inclusive radiative branching ratio $BR(B\to X_s\gamma)$ given by (5), we get

$$BR^{th}(B\to K^*\gamma)=(4.5\pm 1.5)\times 10^{-5}. \tag{43}$$

This branching ratio agrees well with experimental measurement by CLEO [2]

$$BR^{exp}(B\to K^*\gamma)=(4.5\pm 1.5\pm 0.9)\times 10^{-5}.$$

Our numerical analysis has shown that $1/m_b$ corrections in (37)–(39) give rather small contributions to the decay form factor. So in the first crude approximation we can neglect them. This corresponds to the limit $m_b\to\infty$ for $\mathcal{F}_1(\mathbf{\Delta}^2)$. Then the dependence on the Lorentz structure of confining potential is lost and we get a simple formula

$$F_1(0)=\frac{M_B+M_V}{2\sqrt{M_BM_V}}\tilde{\xi}(w), \tag{44}$$

with $w=\frac{M_B^2+M_V^2}{2M_BM_V}$. We have introduced the function

$$\tilde{\xi}(w)=\left(\frac{\beta_V}{\beta}\eta\right)^{3/2}\left(\frac{2}{w+1}\right)^{1/2}\exp\left(-\eta\frac{\tilde{\Lambda}^2}{\beta^2}\frac{w-1}{w+1}\right), \tag{45}$$

which in the limit of infinitely heavy final quark in V meson, coinsides with the Isgur-Wise function of our model [21]:

$$\xi(w)=\left(\frac{2}{w+1}\right)^{1/2}\exp\left(-\frac{\bar{\Lambda}^2}{\beta^2}\frac{w-1}{w+1}\right),\qquad w\equiv v\cdot v'=\frac{M_B^2+M_V^2-q^2}{2M_BM_V}. \tag{46}$$

Thus if we take the formal limit of infinitely heavy s-quark, as it is done in [15], we get for $B\to K^*\gamma$ form factor

$$F_1(0)=\frac{M_B+M_{K^*}}{2\sqrt{M_BM_{K^*}}}\xi(w),\quad \text{for}\ \ m_b\to\infty,\ m_s\to\infty, \tag{47}$$

with $w=\frac{M_B^2+M_{K^*}^2}{2M_BM_{K^*}}$. The relation (47) is, certainly, not adequate, because s-quark is not heavy. It overestimates the form factor approximately by a factor of 1.5.

The approximate formula (44) gives the values of form factors, which are slightly higher than (41). However, the difference does not exceed 10%. Thus we can conclude that the q^2-dependence of form factor F_1 near $q^2=0$ is determined by the function (45).

We can use our results for F_1 to test the HQET relations [30] between the form factors of rare radiative and semileptonic decays of B mesons. Isgur and Wise [30] have shown that in the limit of infinitely heavy b-quark mass an exact relation connects the form factor F_1 with the semileptonic decay form factors defined by:

$$\begin{aligned}\langle V(p_V,e)|\bar{f}\gamma_\mu(1-\gamma_5)b|B(p_B)\rangle &= -(M_B+M_V)A_1(q^2)e^*_\mu\\ &\quad+\frac{2V(q^2)}{M_B+M_V}i\epsilon_{\mu\nu\rho\sigma}e^{*\nu}p_B^\rho p_V^\sigma+\dots\end{aligned} \tag{48}$$

where the ellipses denote terms proportional to $(p_B + p_V)_\mu$ or q_μ. This relation is valid for q^2 values sufficiently close to $q^2_{max} = (M_B - M_V)^2$ and reads:

$$F_1(q^2) = \frac{q^2 + M_B^2 - M_V^2}{2M_B}\frac{V(q^2)}{M_B + M_V} + \frac{M_B + M_V}{2M_B}A_1(q^2). \tag{49}$$

It has been argued in [31,32,14], that in these processes the soft contributions dominate over the hard perturbative ones, and thus the Isgur-Wise relation (49) could be extended to the whole range of q^2.

We can calculate the form factors of semileptonic B-decay in vector light meson at the $q^2 = 0$ point using the method developed here for rare radiative decay form factor $F_1(0)$. It is easy to see that the $1/m_b$ corrections coming from the overlap of meson wave functions are also large. The similar exponential factor as in eq. (32) arises, and we shall not expand it in the $1/m_b$. The $1/m_b$ corrections to the preexponential factor are again rather small. So here we limit our analysis only to the leading order of $1/m_b$ expansion for this factor. In this approximation the semileptonic decay form factors are independent of the Lorentz-properties of the confining $q\bar{q}$-potential and are given [33] by a simple formulae

$$V(0) = \frac{M_B + M_V}{2\sqrt{M_B M_V}}\tilde{\xi}(w), \tag{50}$$

$$A_1(0) = \frac{2\sqrt{M_B M_V}}{M_B + M_V}\frac{1}{2}(1 + w)\tilde{\xi}(w), \tag{51}$$

where $\tilde{\xi}(w)$ is defined by (45). In the limit of infinitely heavy b- and f-quarks the form factors (50) and (51) satisfy all the relations imposed by HQET [8]. The q^2-dependence of the form factors near $q^2 = 0$ is determined for V_1 by the function $\tilde{\xi}(w)$ and for A_1 by the product $\frac{1}{2}(1 + w)\tilde{\xi}(w)$.

It is now easy to check that form factors (50), (51) and (32) satisfy Isgur-Wise relation (49) for the q^2 values near zero. The complete analysis of $1/m_b$ corrections to the semileptonic B-decays into light mesons up to the second order terms will be given elsewhere [33].

7 Conclusions

We have investigated the rare radiative decays of B mesons. The large value ($\sim m_b/2$) of the recoil momentum of the final vector meson requires the completely relativistic treatment of these decays. On the other hand, the presence of large recoil momentum in the energy of the final meson allows for the $1/m_b$ expansion of weak decay matrix element. The contributions to this expansion come both from heavy b-quark mass and large recoil momentum of light vector meson.

Using the quasipotential approach in quantum field theory and the relativistic quark model, we have performed the $1/m_b$ expansion of the rare radiative B-decay form factor $F_1(0)$ up to the second order. We have found that $1/m_b$ corrections coming from the overlap of meson wave functions are large. This agrees with the results of the hybrid sum rules [14], where considerable $1/m_b$ corrections for the form factor $F_1(0)$ have been also obtained. Thus we treat the $\mathbf{\Delta}^2$-dependence of F_1, arising from the overlap of meson wave functions, without expansion. All other $1/m_b$ corrections turn out to be small. They reduce the value of the form factor $F_1(0)$ approximately by 10%. We have calculated the form factors for the decays $B \to K^*\gamma$, $B \to \rho\gamma$ and $B_s \to \phi\gamma$, $B_s \to K^*\gamma$. The found values (41) are in good agreement with the predictions of QCD sum rules [13,14]. Combining our result for $F_1^{B\to K^*\gamma}(0) = 0.32 \pm 0.03$ with the QCD-improved theoretical prediction for inclusive branching ratio (5), we obtain $BR(B \to K^*\gamma) = (4.5 \pm 1.5) \times 10^{-5}$, which is in accord with the experimental branching ratio $BR(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ [2].

We have also evaluated [33] the form factors of B meson semileptonic decays into light vector mesons at the point of maximal recoil. It has been found that the relations between rare and semileptonic decay form factors [30], obtained in the limit of infinitely heavy b-quark, are satisfied in our model.

Aknowledgements

We express our gratitude to B. A. Arbuzov, M. A. Ivanov, J. G. Körner, V. A. Matveev, M. Neubert, V. I. Savrin for the interest in our work and helpful discussions of the results. We are also grateful to A. Yu. Mishurov for the help in numerical calculations and G. G. Likhachev for the help in preparation of this paper for publication. This research was supported in part by the Russian Foundation for Fundamental Research under Grant No.94-02-03300-a and Interregional Centre for Advanced Studies.

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Table caption

Table 1 Comparison of our predictions for the rare radiative decay form factor $F_1(0)$ with light-cone QCD sum rule [13] and hybrid sum rule [14] results.

Figure captions

Fig. 1 Lowest order vertex function

Fig. 2 Vertex function with the account of the quark interaction. Dashed line corresponds to the effective potential (14). Bold line denotes the negative-energy part of quark propagator.

Table 1

Decay	our results	[13]	[14]
$B \to K^*\gamma$	0.32 ± 0.03	0.32 ± 0.05	$0.308 \pm 0.013 \pm 0.036 \pm 0.006$
$B \to \rho\gamma$	0.26 ± 0.03	0.24 ± 0.04	$0.27 \pm 0.011 \pm 0.032$
$B_s \to \phi\gamma$	0.27 ± 0.03	0.29 ± 0.05	
$B_s \to K^*\gamma$	0.23 ± 0.02	0.20 ± 0.04	

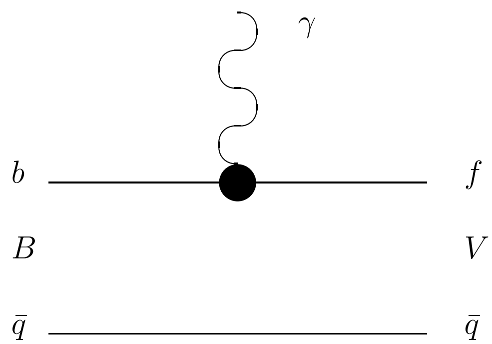
γ
b
f
B
V
$\bar{q}$
$\bar{q}$

Fig. 1

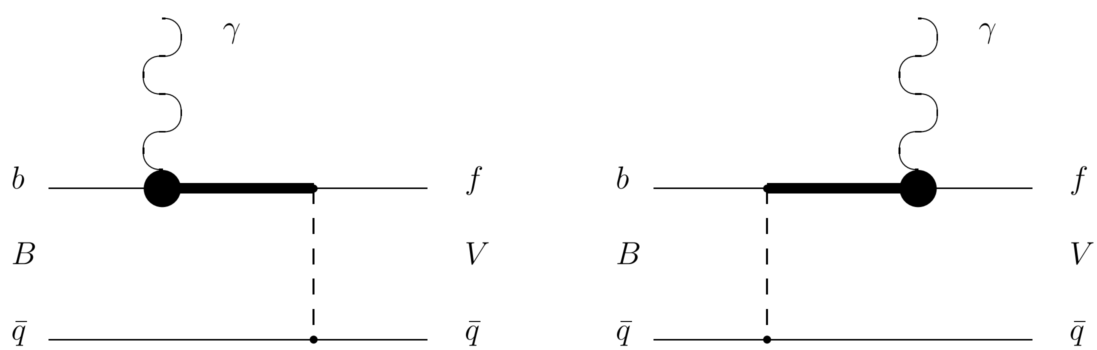
γ
γ
b
f
B
V
$\bar{q}$
$\bar{q}$
b
f
B
V
$\bar{q}$
$\bar{q}$

Fig. 2